**US Army Corps
of Engineers**

Irregular Wave Forces on Heavily Overtopped Thin Vertical Walls

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PURPOSE: The Coastal and Hydraulics Engineering Technical Note (CHETN) described herein provides empirical equations to estimate irregular wave forces and overturning moments on thin, vertical walls extending from the seafloor and having a top elevation that is below the still water level. In this situation the majority of the wave crest passes over the vertical wall. A worked example illustrates application of the empirical equations.

INTRODUCTION AND BACKGROUND: Irregular wave forces on a heavily overtopped thin vertical wall were measured during a series of laboratory experiments at the Coastal and Hydraulics Laboratory. The purpose of the experiments was to obtain site-specific engineering values for New Orleans District to use in design of a current deflection dike located at the mouth of the Mississippi River. Additional tests were run to provide data for developing generic design guidance for heavily overtopped vertical walls. Figure 1 shows the orientation of the current deflection dike, and this configuration was simulated in the experiments. Wall parameters are defined in the Figure 1 cross section. Water depths along the dike ranged up to 21 ft.
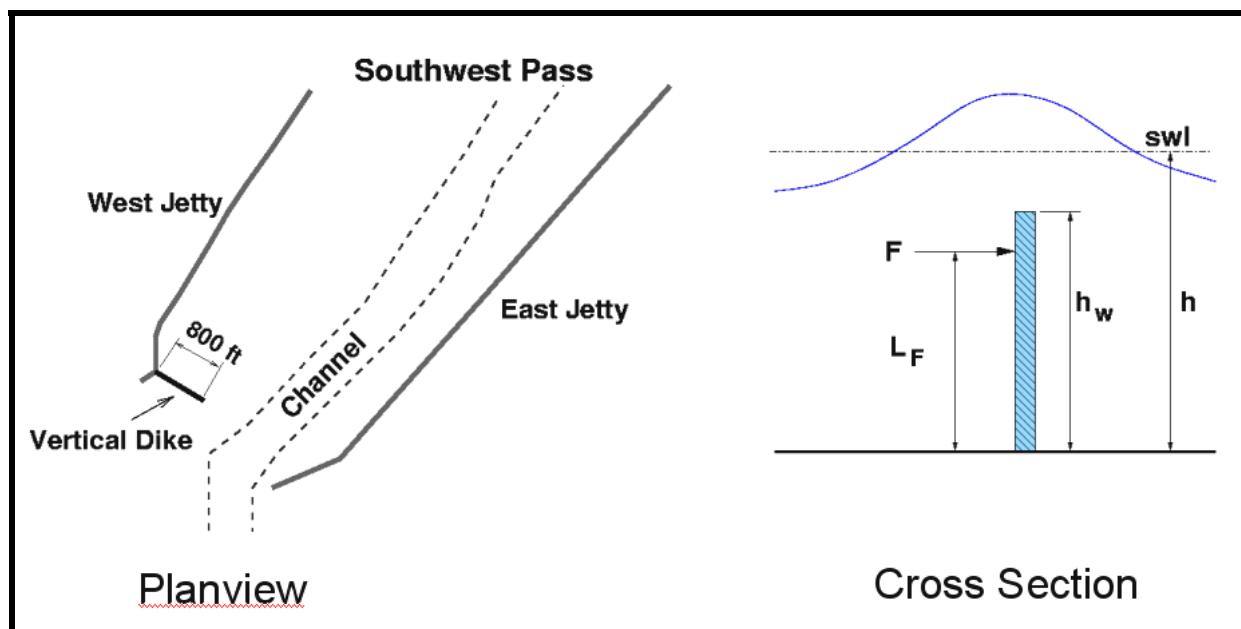

Figure 1. Current deflection dike location at the mouth of the Mississippi River

Constructing the top of the dike at an elevation comparable to the incoming wave trough introduces several hydrodynamic complexities. The dike will be heavily overtopped during storm conditions, and the overflowing water will cause a region of flow separation and lower pressure on the leeside of the wall (Knott and Mackley 1980). This low pressure will increase the shoreward-directed wave force at the top of the wall. Incoming wave characteristics will be altered by the partial wave reflection at the wall.

MEASUREMENTS AND ANALYSES: Experiments were conducted in a large basin at a geometrically undistorted length scale of 1:50. Water elevation was held constant giving a water depth of 23.8 ft (prototype scale) for all experiments. Four wall heights were tested having top elevations relative to still water level of -7.8 ft, -4.8 ft, -1.8 ft, and +1.2 ft, respectively. Irregular wave conditions were near depth-limited breaking at the wall for many of the tests. Zeroth-moment significant wave heights (H_{mo}) varied between 5 ft and 12 ft (prototype scale), and the wave period associated with the peak of the wave spectrum (T_p) varied between 7.0 and 13.5 sec (prototype scale).

The key measurements of these experiments were the incoming waves and the resultant forces on the overtopped vertical wall. Wave forces on the vertical dike were measured using the apparatus shown in the center photograph in Figure 2. This force-measuring portion is the cantilevered wall section supported by the vertical framework. Narrow gaps separate the supported wall section from the adjacent fixed wall. Wave forces applied over the wall section result in reactions at the upper supports as illustrated by the free-body diagram in Figure 2. Two force transducers were used at the fulcrum point (F_2 and F_3) and a third transducer was used at the top (F_1). Analysis of the free-body diagram at any time yields the total wave force F_L and the corresponding moment arm L_F about the wall base at that instant. Calibration was performed using the apparatus shown on the right-hand side of Figure 2.
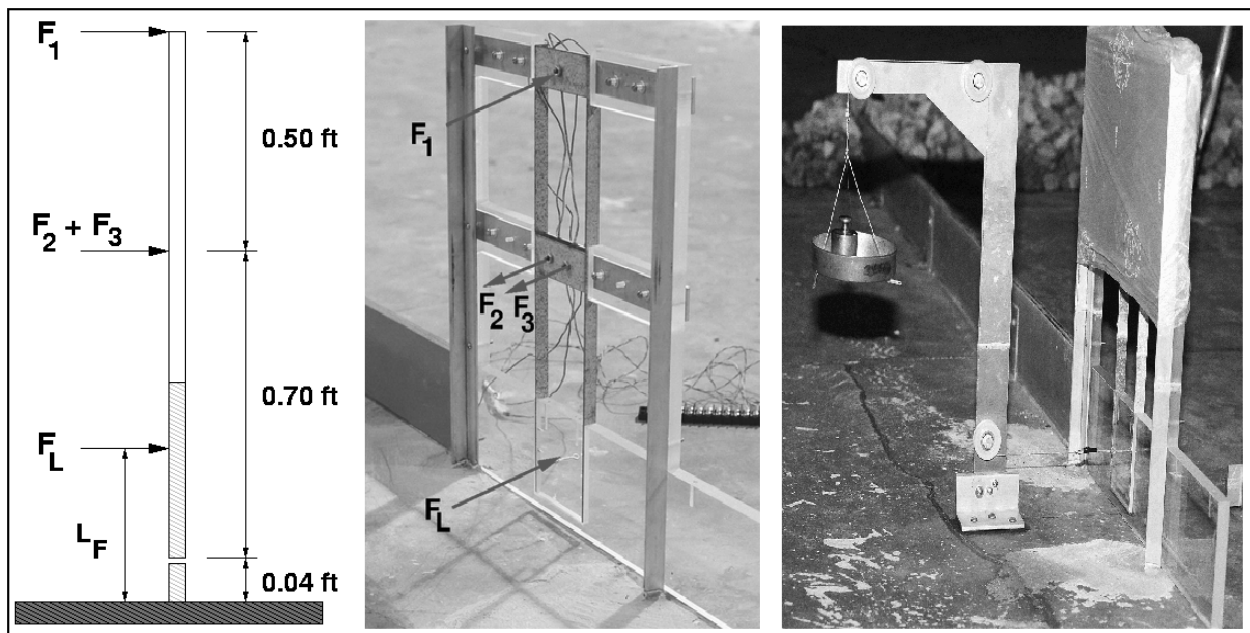

Figure 2. Force measuring section of model vertical dike

Force data were collected from the three load cells at a 40-Hz rate. This logging rate was sufficient for recording pulsating wave loads, but the rate was not high enough to record impact loads. Because the top elevation of the wall was well beneath the wave crest elevation, impact loading was much less probable than for emergent walls.

The three synchronous force time series from the load cells were combined at each time step according to the force balance equations derived from the free-body diagram of Figure 2. This resulted in time series of the total force on the force-measuring section (F_L) and the corresponding moment arm about the seabed (L_F). Measurements were converted to prototype size prototype-scale force per unit wall length. The force time series exhibited characteristic sharp peaks in the shoreward direction corresponding to the wave crests, and broad lower peaks in the seaward direction resulting from the passage of the wave trough over the wall.

For each time series the shoreward-directed and seaward-directed peak forces were extracted and plotted as distributions normalized by the root-mean-square of the peak forces (F_{rms}) for the time series. Similarly, the distributions of shoreward- and seaward-directed peak moments were determined as the product of peak force and corresponding moment arm. Forces and moments were always larger in the shoreward direction corresponding to the passage of wave crests, and magnitudes increased with higher wall top elevation, larger zeroth-moment wave height, and longer peak wave periods. Figure 3 shows typical results of the force and moment distributions.
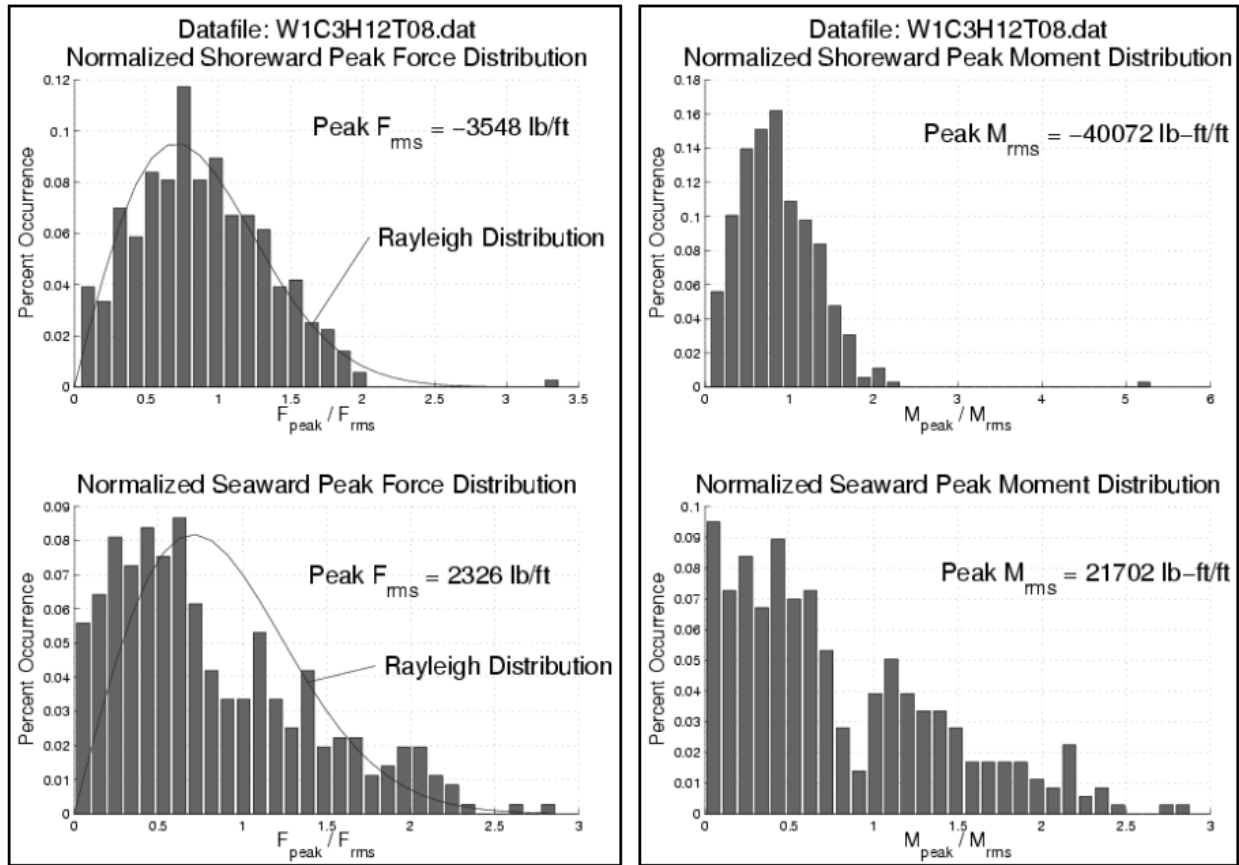

Figure 3. Representative distributions of shoreward and seaward peak forces and moments

The solid curve on the force distributions in Figure 3 is the Rayleigh distribution based on the value of F_{rms} for the force peak distribution. The shoreward-directed peak force distribution is well represented by the Rayleigh distribution whereas the seaward-directed force distribution is a poorer match. More detailed descriptions of the measurements and analyses are given in Hughes, Winer, and Blodgett (2006).

RAYLEIGH DISTRIBUTION OF SHOREWARD-DIRECTED FORCES: Figure 4 compares actual shoreward-directed peak force distribution parameters $F_{1/3}$ and $F_{1/10}$ to estimates using the Rayleigh distribution based on F_{rms} (where $F_{1/3}$ and $F_{1/10}$ are the average of the highest 1/3 and highest 1/10 of the force peaks, respectively). Estimates of $F_{1/3}$ were quite good with little scatter around the line of equivalence. The most variation was for the largest forces observed at the wall with top elevation +1.2 ft (prototype scale) above the still water level. More scatter was seen for estimates of $F_{1/10}$ as shown on the right side plot in Figure 4; however, the variation is not large, and it appears to be evenly distributed about the line of equivalence.
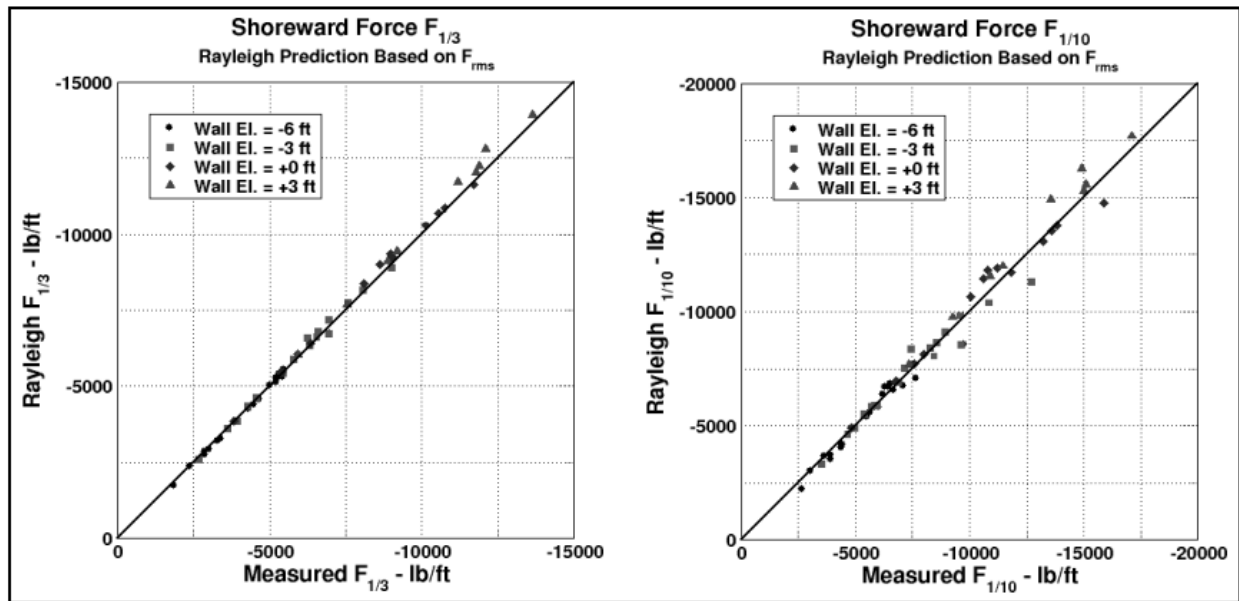

Figure 4. Shoreward-directed peak force prediction based on Rayleigh distribution

Given the good correspondence between measurements and prediction using the Rayleigh distribution, design guidance presented in this Technical Note is based on estimating the shoreward-directed root-mean-squared force F_{rms}, and then using the Rayleigh distribution to estimate extreme peak forces for the wave condition. The relationship between F_{rms} and other statistical peak force parameters conforming to the Rayleigh distribution are given by the following formulas.

$$F_{1/3} = 1.416\, F_{rms} \tag{1}$$

$$F_{1/10} = 1.80\, F_{rms} \tag{2}$$

$$F_{1/100} = 2.36\, F_{rms} \tag{3}$$

$$F_{1/250} = 2.55\, F_{rms} \tag{4}$$

4

PREDICTION OF SHOREWARD-DIRECTED F_{rms} PEAK FORCE: The force parameter representing the root-mean-squared values of the shoreward-directed peak forces increased with increasing top elevation of the vertical wall, and relative wall height (h_w/h) appears to the be most influential parameter. The F_{rms} peak force also increased gradually as the peak spectral wave period increased (decreasing relative water depth). Most experiments were conducted with waves approaching depth-limited breaking, and the RMS force increased as the wave height increased. However, experiments conducted with lower wave heights tended to introduce more scatter into the results. This may have been caused by decreased flow separation as the wave crest passed over the wall.

A theoretical analysis of slightly submerged vertical walls subject to overtopping waves suggested that the parameter $F_o (h_w/h)^2$ might be a good normalizing factor for F_{rms}, where h_w is height of the wall above the bottom, h is water depth, and F_o is a characteristic force proportional to the peak wave force acting on a fully emergent wall. There are several ways to estimate the value of F_o. Three methods were considered:

  F_o is proportional to horizontal force estimated from linear wave theory
  F_o is proportional to horizontal force estimated using Goda's (1974) method
  F_o is proportional to the total nonlinear wave momentum flux at the wave crest

All three estimates for F_o in the normalizing factor gave reasonable results. However, estimates of F_o based on linear wave theory and calculated using the Goda method resulted in a normalized peak RMS force that still exhibited an increasing trend with increasing wave period. This is probably related to increasing wave nonlinearity that is not captured by the first two methods. However, the wave nonlinearity was better represented when F_o was assumed to be proportional to total nonlinear wave momentum flux, M_F.

Hughes (2003, 2004) presented empirical equations to estimate the total nonlinear depth-integrated wave momentum flux at the wave crest in terms of the relative wave height and relative water depth. For irregular waves the following formula for M_F was recommended.

$$\left(\frac{M_F}{\rho g h^2} \right)_{max} = A_0 \left(\frac{h}{g T_p^2} \right)^{-A_1} \tag{5}$$

where

$$A_0 = 0.639 \left(\frac{H_{mo}}{h} \right)^{2.026} \tag{6}$$

$$A_1 = 0.180 \left(\frac{H_{mo}}{h} \right)^{-0.391} \tag{7}$$

and H_{mo} is zeroth-moment wave height, T_p is peak spectral wave period, h is water depth, ρ is water density, and g is gravitational acceleration.

Figure 5 presents the shoreward-directed normalized root-mean-squared peak wave force versus relative water depth. The relative wall height parameter accounts for much of the scatter reduction, and the nonlinear wave momentum flux parameter seems to have accounted for wave nonlinearities because the data do not exhibit an increase with wave period (decreasing relative depth). The most scatter occurs at the wave period where additional tests were conducted with smaller wave heights.
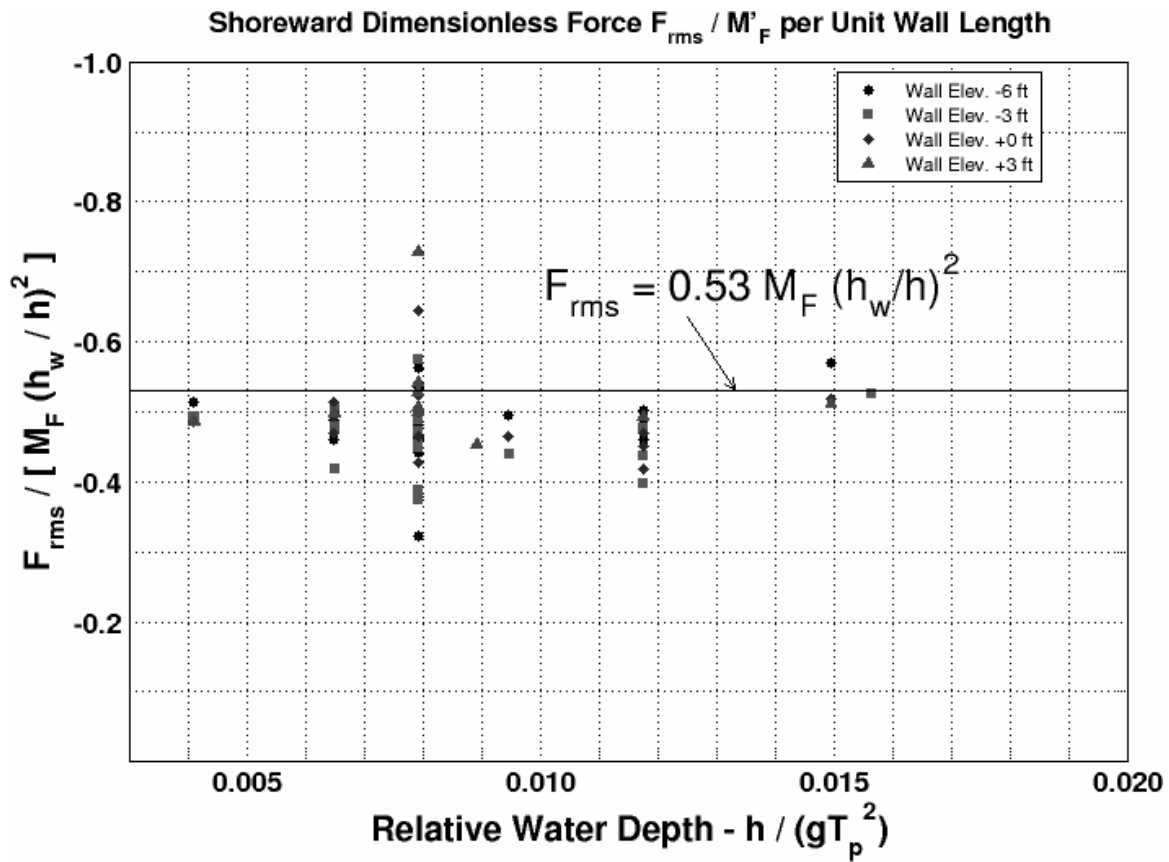
Figure 5. Shoreward-directed normalized RMS peak wave force versus relative water depth

The solid horizontal line in Figure 5 was drawn as a conservative recommendation for estimating the shoreward-directed RMS peak wave force acting on the overtopped vertical wall. This resulted in the following simple equation for estimating F_{rms} (per unit length of wall)

$$F_{rms} = 0.53 M_F \left(\frac{h_w}{h} \right)^2 \tag{8}$$

where M_F is calculated using the formulas given in Eqs. 5-7. Because most of the experiments were conducted with waves approaching the depth-limiting condition, Eq. 8 may not be appropriate for smaller waves in deeper water. Figure 6 compares measured total peak shoreward-directed F_{rms} force to predictions based on Eq. 8. Because the predictive equation was purposely conservative, nearly all of the predictions are greater than actual measurements and fall below the line of equivalence.
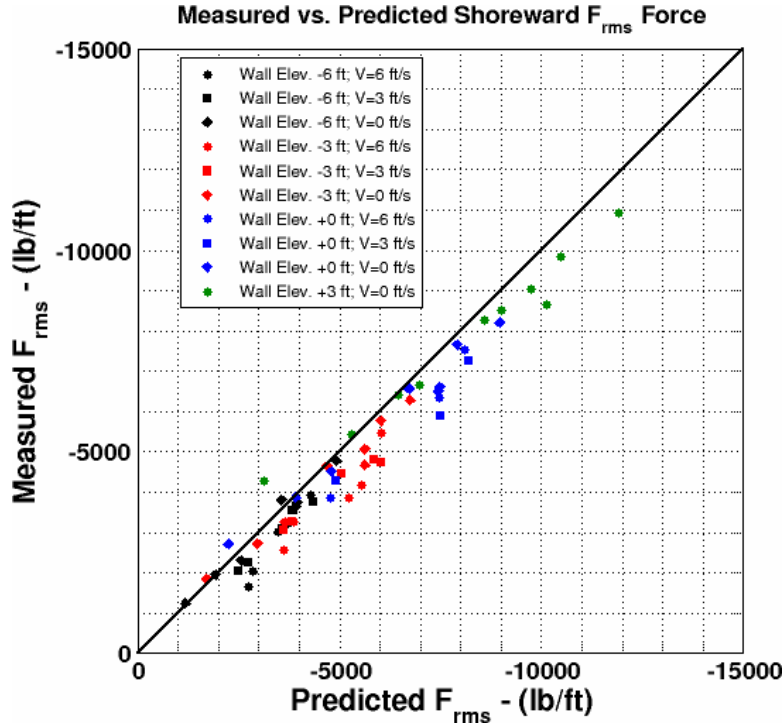

Figure 6. Measured versus predicted shoreward-directed total peak F_{rms} force

PREDICTION OF MOMENT ARM ASSOCIATED WITH TOTAL PEAK FORCE: The moment arm is defined as the vertical distance from the sea floor to the location of the total peak force as shown in the cross section of Figure 1 and the free-body diagram of Figure 2. For each experiment the variation of calculated moment arm L_F associated with the peak shoreward-directed wave forces was examined. The moment arm was reasonably constant over most of the peak force distribution range, so an average value was selected for each experiment. An appropriate normalizing factor was found by trial and error that yielded a strictly empirical formula giving a conservative estimate of the moment arm as a function of water depth, wall height, and peak wave period, i.e.,

$$L_F = 0.4\, h_w \sqrt{\frac{h}{h_w}} \left(\frac{h}{g\, T_p^{\,2}} \right)^{-0.1} \tag{9}$$

7

The moment about the base of the vertical wall per unit wall length is estimated as the product of the total peak wave force times the moment arm.

$$M = F \cdot L_F \tag{10}$$

For example, the moment associated with the RMS force is given as

$$M_{rms} = F_{rms} \cdot L_F \tag{11}$$

Figure 7 compares measured shoreward-directed RMS moments to predictions based on Eqs. 8, 9, and 11. The conservative nature of the moment predictions is evident with most points falling below the line of equivalence.
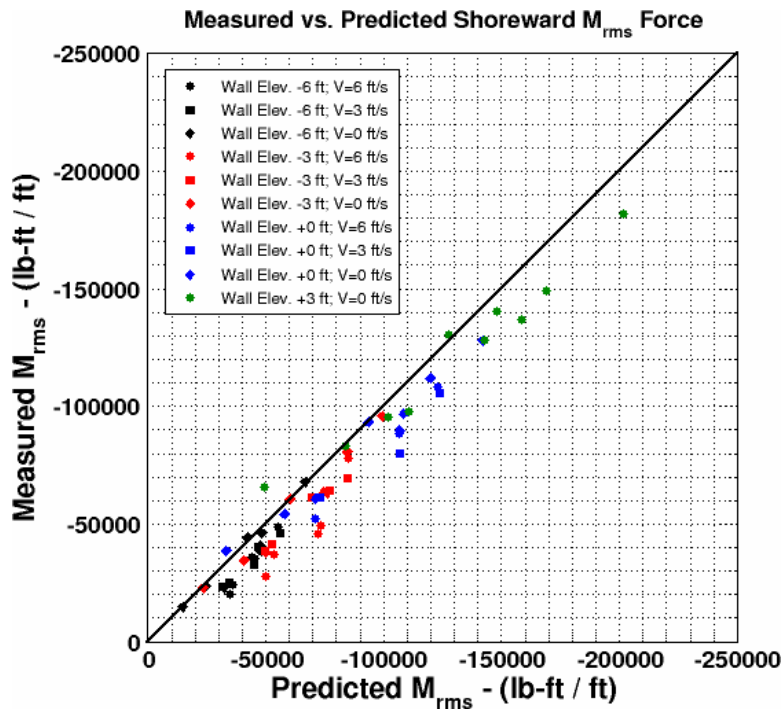

Figure 7. Measured versus predicted shoreward-directed total peak M_{rms} moment

Example: Irregular Wave Force on Overtopped Thin Wall

Find: The total peak shoreward-directed forces F_{rms}, $F_{1/3}$, $F_{1/10}$, $F_{1/100}$, $F_{1/250}$, and the corresponding moments acting about the base of the thin vertical wall shown in Figure 8.
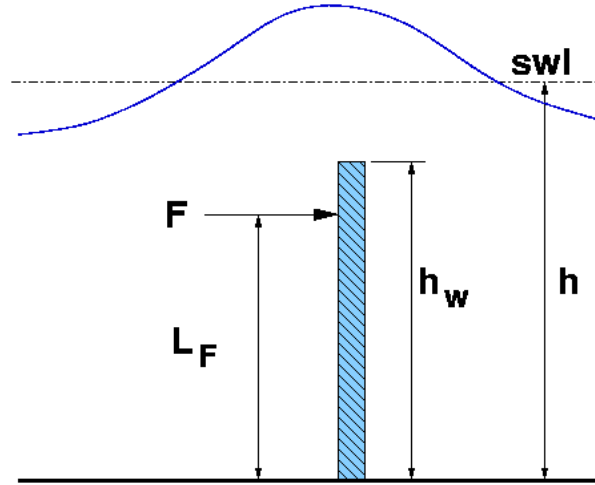

Figure 8. Overtopped wall parameters

Given:

h	=	20 ft	– Water depth
h_w	=	18 ft	– Wall height
T_p	=	9 sec	– Wave period associated with the spectral peak
H_{mo}	=	8 ft	– Zeroth-moment significant wave height
g	=	32.2 ft/sec^2	– Gravitational acceleration
ρg	=	64.0 lb/ft^3	– Specific weight of sea water

Calculate the Wave Momentum Flux Parameter: First calculate values of relative wave height and relative depth as

$$\frac{H_{mo}}{h} = \frac{8\,ft}{20\,ft} = 0.4 \qquad \text{and} \qquad \frac{h}{g\,T_p^{\,2}} = \frac{20\ ft}{(32.2\ ft/\sec^2)(9\ \sec)^2} = 0.0077$$

Next, find the values of the coefficient A_0 and A_1 from Eqs. 6 and 7, respectively, i.e.,

$$A_0 = 0.639\left(\frac{H_{mo}}{h}\right)^{2.026} = 0.639\,(0.4)^{2.026} = 0.0998$$

9

$$A_1 = 0.180 \left(\frac{H_{mo}}{h} \right)^{-0.391} = 0.180 \, (0.4)^{-0.391} = 0.2576$$

The nondimensional wave momentum flux parameter is calculated from Eq. 5 as

$$\left(\frac{M_F}{\rho g h^2} \right)_{max} = A_0 \left(\frac{h}{g T_p^{\,2}} \right)^{-A_1} = 0.0998 \, (0.0077)^{-0.2576} = 0.35$$

Finally, the dimensional wave momentum flux is determined to be

$$M_F = 0.35 \, \rho g h^2 = 0.35 \, (64.0 \, lb/ft^3) \, (20 \ ft)^2 = 8{,}960 \, lb/ft$$

Calculate the RMS Force F_{rms}:

Using Eq. 8 the root-mean-squared shoreward-directed total peak force per unit length of vertical wall is found as

$$F_{rms} = 0.53 \, M_F \left(\frac{h_w}{h} \right)^2 = 0.53 \, (8{,}960 \, lb/ft) \left(\frac{18 ft}{20 ft} \right)^2 = 3{,}847 \, lb/ft$$

Calculate $F_{1/3}$, $F_{1/10}$, $F_{1/100}$, and $F_{1/250}$ Using the Rayleigh Distribution:

Equations 1-4 give the appropriate relationships for the Rayleigh distribution. Note that other representative values can also be specified by the Rayleigh distribution. Substituting the value for F_{rms} yields

$$F_{1/3} = 1.416 \, F_{rms} = 1.416 \, (3{,}847 \, lb/ft) = 5{,}447 \, lb/ft$$
$$F_{1/10} = 1.80 \, F_{rms} = 1.80 \, (3{,}847 \, lb/ft) = 6{,}925 \, lb/ft$$
$$F_{1/100} = 2.36 \, F_{rms} = 2.36 \, (3{,}847 \, lb/ft) = 9{,}079 \, lb/ft$$
$$F_{1/250} = 2.55 \, F_{rms} = 2.55 \, (3{,}847 \, lb/ft) = 9{,}810 \, lb/ft$$

Calculate Moments Associated with the Forces $F_{1/3}$, $F_{1/10}$, $F_{1/100}$, and $F_{1/250}$:

First estimate the moment arm for this particular case using Eq. 9, i.e.,

$$L_F = 0.4 \, h_w \sqrt{\frac{h}{h_w}} \left(\frac{h}{g T_p^{\,2}} \right)^{-0.1} = 0.4 \, (18 ft) \sqrt{\frac{20 ft}{18 ft}} \, (0.0077)^{-0.1} = 12.35 \, ft$$

Finally, the associated moments per unit wall length about the base of the wall are found by substituting the calculated moment arm and various calculated representative peak forces into Eq. 10 yielding…

$$M_{rms} = F_{rms} \cdot L_F = (3{,}847 \; lb/ft) \, (12.35 \, ft) = 47{,}510 \; (lb-ft)/ft$$

$$M_{1/3} = F_{1/3} \cdot L_F = (5{,}447 \; lb/ft) \, (12.35 \, ft) = 67{,}270 \; (lb-ft)/ft$$

$$M_{1/10} = F_{1/10} \cdot L_F = (6{,}925 \; lb/ft) \, (12.35 \, ft) = 85{,}524 \; (lb-ft)/ft$$

$$M_{1/100} = F_{1/100} \cdot L_F = (9{,}079 \; lb/ft) \, (12.35 \, ft) = 112{,}126 \; (lb-ft)/ft$$

$$M_{1/250} = F_{1/250} \cdot L_F = (9{,}810 \; lb/ft) \, (12.35 \, ft) = 121{,}154 \; (lb-ft)/ft$$

Remarks: The estimates of peak forces and moments per unit length of vertical wall calculated in this example are conservative. The seaward-directed forces and moments will be less than the shoreward-directed values given by the design guidance in this technical note. The guidance is appropriate only for vertical walls where the top elevation is between the still water level and an elevation at or slightly below the wave trough elevation. The tested range of relative wall height was $0.67 \leq h_w/h \leq 1.05$. Forces on walls with greater submergence (values of h_w/h less than 0.67) may not be correctly estimated using the formulas given in this technical note.

SUMMARY: This CHETN has described a method for estimating total peak wave forces and moments on thin vertical walls that experience heavy wave overtopping. Top elevation of the vertical wall is assumed to be somewhere in the upper 1/3 of the water column. Laboratory measurements of wave forces on heavily overtopped vertical walls showed that shoreward-directed forces were the largest, and the data indicated the distribution of the force peaks is well represented by the Rayleigh distribution based on the root-mean-squared force. An empirical equation (Eq. 8) is presented in terms of relative wall height and wave momentum flux to estimate the shoreward-directed RMS peak force. The associated moment arm, which is nearly constant over most of the peak force distribution, is also given by an empirical equation (Eq. 9). For design application, first estimate the RMS peak force, then use the Rayleigh distribution (Eqs. 1-4) to obtain an appropriate design force (e.g., $F_{1/100}$), and finally estimate the corresponding moment as the product of the force and moment arm using Eq. 10.

ADDITIONAL INFORMATION: This CHETN is a product of the Coastal Structures Asset Management Work Unit of the Coastal Inlets Research Program (CIRP) being conducted at the U.S. Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory. Questions about this technical note can be addressed to Dr. Steven A. Hughes (Voice: 601-634-2026, Fax: 601-634-3433, email: Steven.A.Hughes@erdc.usace.army.mil). For information about the Coastal Inlets Research Program (CIRP), please contact the CIRP Program Manager, Dr. Nicholas C. Kraus at 601-634-2016 or at Nicholas.C.Kraus@erdc.usace.army.mil. Beneficial reviews were provided by Mr. Edward Blodgett, U.S. Army Engineer District, New Orleans, and Dr. David Kriebel, U.S. Naval Academy.

This document should be cited as:

Hughes, S. A. (2006). "Irregular wave forces on heavily overtopped thin vertical walls," Coastal and Hydraulics Engineering Technical Note ERDC/CHL CHETN-III-__, U.S. Army Engineer Research and Development Center, Vicksburg, MS.

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